Algebraic geometry 1

Exercise Sheet 9

PD Dr. Maksim Zhykhovich Winter Semester 2025, 29.12.2025

Exercise 1. Let $F, G \in K[X_0, X_1, X_2]$ be two non-constant homogeneous polynomials. Show that $V^p(F, G) \neq \emptyset$ in $\mathbb{P}^2$.

Exercise 2. (1) Show that any morphism from $\mathbb{P}^2$ to $\mathbb{P}^1$ is a constant map.
Hint: Use Exercise 1, Sheet 8, and Exercise 1 above.
(2) Show that $\mathbb{P}^2$ and $\mathbb{P}^1 \times \mathbb{P}^1$ are not isomorphic.

Exercise 3. Let X be a topological space and let $A, B \subset X$ be subsets.
(1) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
(2) Show that A is irreducible if and only if $\overline{A}$ is irreducible.
(3) Show that $\dim Y \leq \dim X$ for any subset $Y \subset X$.
(4) Assume that X is irreducible and finite-dimensional. Let Y be a closed subset of X, such that $\dim Y = \dim X$. Show that $X = Y$.

Exercise 4. (1) Let X be a topological space and let $X = \cup_i U_i$ be an open covering. Show that $\dim X = \sup_i \{\dim U_i\}$.
(2) Deduce from (1) that $\dim \mathbb{P}^n = n$.
Hint: We proved in the lecture that $\dim \mathbb{A}^n = n$.